Modelling the dynamics of infinitesimal particles orbiting non-spherical rotating bodies

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Overview

1. The orbital environment of minor bodies
   - Motivations for studying the orbital dynamics around minor bodies
   - The orbital dynamics environment of minor bodies
   - Perturbations
2. Dynamics modelling
   - Cartesian equations of motion within the minor body-centred reference frame
   - Hamiltonian approach
   - Tools
3. Stability analysis
   - Ground-track resonances
   - Secular resonances
   - Long-term variation of the inclination of distant circular orbits

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Motivations for studying the orbital dynamics around minor bodies

• The study of orbital evolution around an oblate object is capturing a renewed focus with the recent and forthcoming *"in situ" explorations*.

• The missions designed to orbit and land on the surface of minor bodies, asteroids or comets, raise *new open questions* and stimulate the development of *new lines of research*. The planning of close-proximity operations, the computation of stable spacecraft trajectories about a small object, the landing on a rotating body rely on a *good knowledge of the dynamics around the explored object*.

• A cartographic study of the dynamics around minor bodies might assess to the possibility of existence of accompanying swarms of particles.

• The mechanics of the motion in the neighborhood of a minor body depends on many factors and parameters. Observations, studies and missions to asteroids and comets, such as DAWN, OSIRIS-REx, HAYABUSA, HAYABUSA2, NEAR SHOEMAKER, ROSETTA, STARDUST, DEEP SPACE 1, CASSINI, GALILEO, DART, revealed that explored objects are characterized by a variety of *shapes, sizes, densities, morphologies, spin states, complex environments, forces and perturbations acting in their close environments.*

The orbital dynamics environment of minor bodies

- The wide variety of dynamical behaviors around oblate bodies, such as *escape, impact and capture orbits, the long-term stability of moonlets around asteroids, the complex effects of resonances* are just some examples showing the compelling need for investigating the global dynamics in the proximity of small objects.

- Given *the complex mechanics* of such systems, in which many parameters are unknown, on one hand, and *the variety of dynamical phenomena* that occur on short, medium and large time scales, on the other hand, both the dynamics modelling and the stability analysis are challenging.

Perturbing forces

- The motion around an oblate minor body is influenced by gravitational and non-gravitational perturbations:

– attraction of the oblate minor body and third body perturbations (attraction of the Sun, planets and possible satellites of the minor body);

– the solar radiation pressure, comet outgassing pressure, YORP effect which leads to changes of the rotation state of the oblate body.

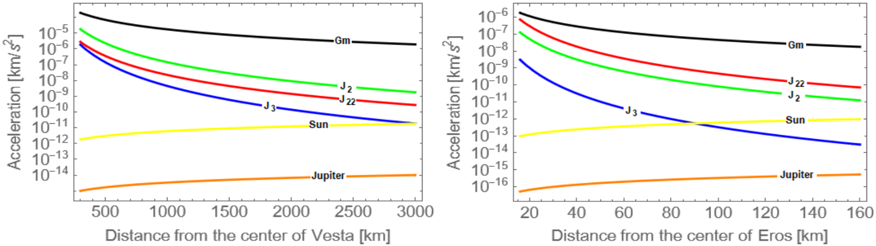

Figure 1: Left: Order of magnitude of various perturbations in the case of Vesta (left) and Eros (right).

Aim of our study

• Motivated by some recent results we obtained in characterizing the Earth's orbital environment, we are developing some models and tools which might be applied in studying the dynamics in strongly perturbed environments, at least with an altitude from the body surface.

• In **modelling** the orbital evolution around oblate minor bodies we are interested in:
- assess the key parameters and the appropriate approach for making the modeling process as generic as possible;
- Cartesian approach (the centred body system): propagate keplerian orbits perturbed by the Sun and the oblateness of attracting body (asteroid or comet). The spherical harmonics approach is used and the coefficients are assumed to be known;
- Hamiltonian formalism: classify secular, resonant, and short periodic terms; classify and study the effects of resonances; Use various expansions, in terms of the orbital elements (or canonical variables), for the disturbing function due to the Sun;

• In **Stability analysis:**
- the evolution of the orbital elements on short, medium and long time scales;
- identify the key role played by each disturbing term;
- determination and study of equilibria;
- study of the effects of resonances (ground-track or tesseral resonances and secular resonances);

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Equations of motion in Cartesian coordinates

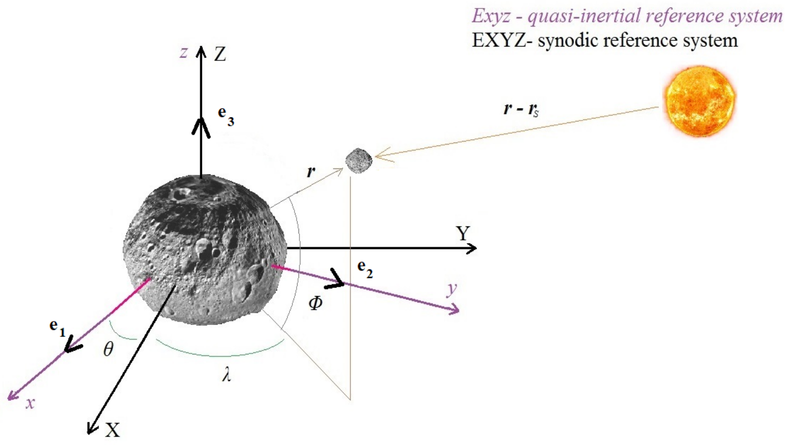

Figure 2: Quasi–inertial and synodic reference systems.

- The motion is described by:

$$\ddot{\mathbf{r}} = R_3(-\theta)\,\nabla V(\mathbf{r}) - Gm_S\left(\frac{\mathbf{r}-\mathbf{r}_S}{|\mathbf{r}-\mathbf{r}_S|^3} + \frac{\mathbf{r}_S}{|\mathbf{r}_S|^3}\right) + \mathbf{a}_{ng}\ , \tag{1}$$

- In terms of spherical harmonics, the gravity potential has the form (Kaula 1966):

$$V(r,\phi,\lambda) = -\sum_{n=0}^{\infty}\sum_{m=0}^{n} V_{nm} = \frac{Gm_A}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_A}{r}\right)^n P_{nm}(\sin\phi)\ J_{nm}\cos[(m\lambda-\lambda_{nm})]\ ,$$

- The fast Lyapunov indicator (hereafter FLI), (see (C. Froeschlé et al. 1997, Guzzo et al. 2002, 2013) defined as:

$$FLI(\mathfrak{X}(0),\mathcal{V}(0),T) \equiv \sup_{0<t\leq T}\log||\mathcal{V}(t)||\ ,$$

for an initial condition $\mathfrak{X}(0)$ at a time $t = T$, provides information on the regular or chaotic character of the dynamics, the location of the equilibrium points, the role of higher degree harmonic terms. Here, $\mathfrak{X}$ is the state of the system and $\mathcal{V}$ is the tangent vector.

Hamiltonian approach

• Using the action–angle Delaunay variables $(L, G, H, M, \omega, \Omega)$, which are related to the orbital elements $(a, e, i, M, \omega, \Omega)$ by

$$L = \sqrt{\mu_A a} \ , \qquad G = L\sqrt{1-e^2} \ , \qquad H = G \cos i \ ,$$

we are getting the canonical equations of motion associated to the Hamiltonian

$$\mathcal{H} = -\frac{(Gm_A)^2}{2L^2} + \mathcal{H}_A + \mathcal{H}_{Sun} + \mathcal{H}_{SRP} \ ,$$

where

$$\mathcal{H}_A = -\mathcal{R}_A \,, \quad \mathcal{H}_{Sun} = -\mathcal{R}_{Sun} \,, \quad \mathcal{H}_{SRP} = -\mathcal{R}_{SRP} \,,$$

and are the $\mathcal{R}_A$, $\mathcal{R}_{Sun}$ and $\mathcal{R}_{SRP}$ are the disturbing functions.

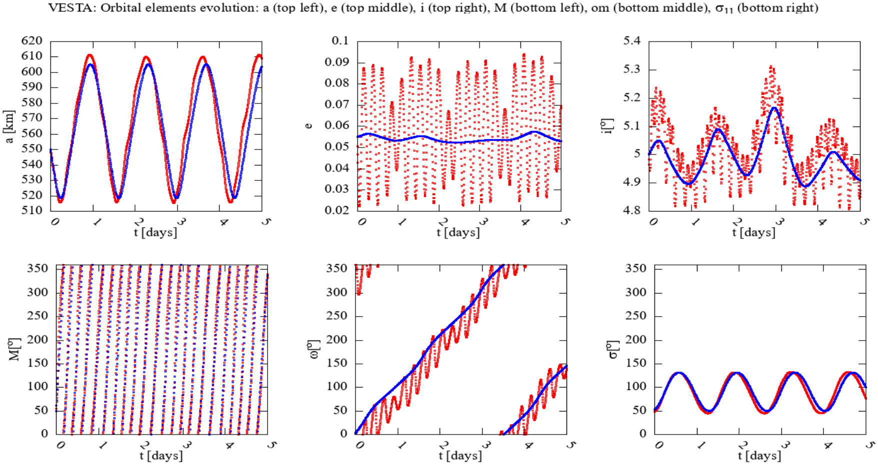

Figure 3: Evolution of the orbital elements using the Cartesian equations (red and green) and Hamiltonian approach (blue), under the effects of the Sun, J_2 and J_3 (green and blue), the Sun, J_2, J_3, J_{22}, J_{31}, J_{32}, J_{33} (red).

Tools

•We developed **two generic programs**, which can be adapted to any body, provided the following key parameters of the minor body are given: the orbital elements, the ecliptic coordinates of the north celestial pole, the rotation period and the spherical harmonics coefficients of the minor body.

• First program implements the Cartesian equations of motion and takes into account the attraction of the Sun, and the harmonics: J_2, J_3, J_4, J_{22}, J_{31}, J_{32}, J_{33};

• The second program deals with the single averaged Hamiltonian. The disturbing functions $\mathcal{R}_A$, $\mathcal{R}_{Sun}$ and $\mathcal{R}_{SRP}$ are averaged over the mean motion of the satellite and all the ground-track resonant terms, up to degree and order $n = m = 4$, for the resonances $1:1$, $2:1$, $3:1$, $3:2$, $4:1$, $4:3$, $1:2$, $1:3$, $2:3$, are taken into account.

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Ground-track resonances

- *Ground-track (or tesseral) resonances*, occur whenever the orbital period of the satellite and the rotation of the minor body are commensurable, i.e. $\ell \dot{M} - j\dot{\theta} = 0$, $\ell, j \in \mathbb{N}$, is satisfied. As effect of tesseral resonances, the semi–major axis varies on a time scale of the order of a few orbital revolutions of the satellite.
- One of the key parameters defining the location of ground-track resonances is the rotation period of the minor body. For fast rotators, the ground-track resonances are located very close to their surface or even inside them.

- The second program computes also the FLI and a cartographic study of the phase space.

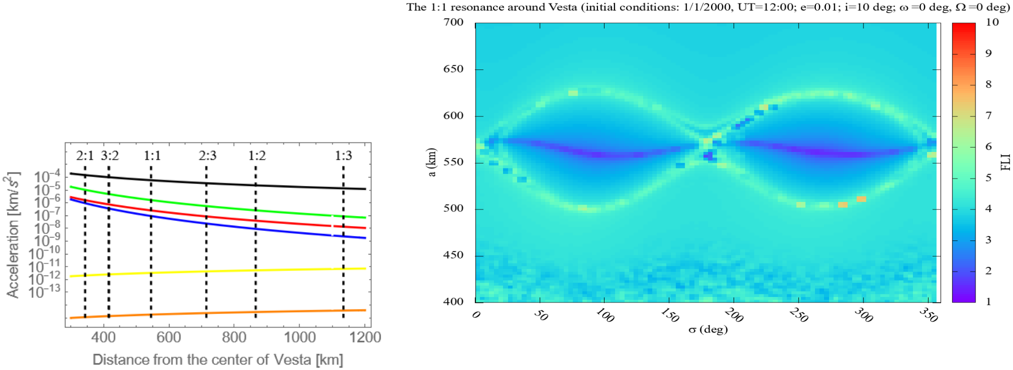


Figure 4: Vesta: Location of tesseral resonances (left); Cartography of the 1:1 resonance (Hamiltonian approach) (right).

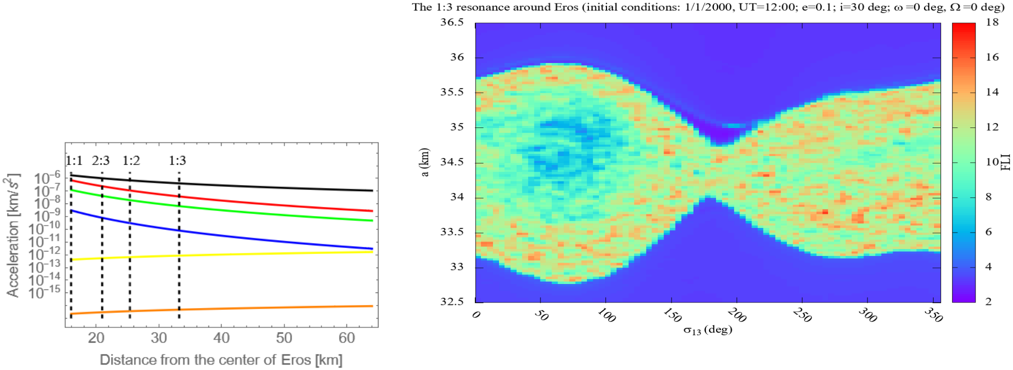


Figure 5: Eros: Location of tesseral resonances (left); Cartography of the 1:3 resonance (Hamiltonian approach) (right).

Secular resonances

• Secular resonances occur when $k_1\dot{\omega} + k_2\dot{\Omega} + k_3\dot{M}^* = 0, \quad (k_1, k_2, k_3) \in \mathbb{Z}^3$. Their effects are the long-term variation of eccentricity and inclination on time scales of the order of tens (or hundreds) of revolutions.

• Since in the vicinity of the minor body the acceleration induced by J_2 is larger than that induced by the Sun, at least in these regions of the space $\dot{\omega}$ and $\dot{\Omega}$ could be approximated as:

$$\dot{\omega} \simeq \frac{3J_2R_A^2}{4}\frac{\mu_A^{1/2}}{a^{7/2}}(1-e^2)^{-2}(5\cos^2 i - 1) \quad rad/day \,,$$

$$\dot{\Omega} \simeq -\frac{3J_2R_A^2}{2}\frac{\mu_A^{1/2}}{a^{7/2}}(1-e^2)^{-2}\cos i \quad rad/day \,.$$

• In consequence, in the neighborhood of a minor body, we can classify the resonances as:

i) resonances depending only on the inclination: $\alpha\dot{\omega} + \beta\dot{\Omega} = 0$, with $\alpha, \beta \in \{\pm 2, \pm 1, 0\}$.

ii) resonances depending on a, e and i: $\alpha\dot{\omega} + \beta\dot{\Omega} + \gamma\dot{M}^* = 0$, with $\alpha, \beta \in \{\pm 2, \pm 1, 0\}$ and $\gamma \neq 0$.

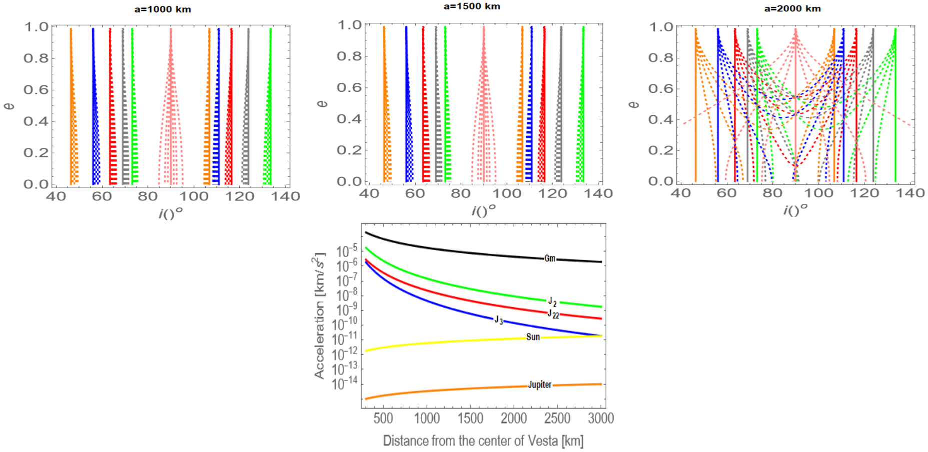


Figure 6: Vesta: The web like structure of the secular resonances at $a = 1000$ km, $a = 1500$ km, $a = 2000$ km.

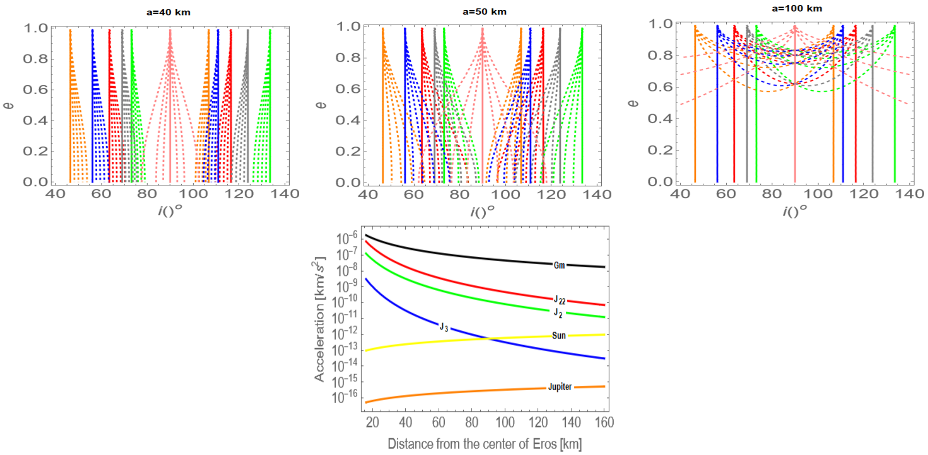


Figure 7: Eros: The web like structure of the secular resonances at $a = 40$ km, $a = 50$ km, $a = 100$ km.

Long-term variation of the inclination of distant circular orbits

- Following Allan and Cook 1964, who studied distant circular orbits of Earth's satellites, we considered a double averaged model that includes the effects of J_2 and the attraction of the Sun in order to analyse the long-term variation of the inclination of distant circular orbits around minor bodies.
- Expressed in the Milankovitch elements $\mathbf{e} = e\mathbf{P}$, $h = (1-e)^2\mathbf{R}$, where $\mathbf{P}$ is a unit vector in the orbital plane and directed toward pericentre, while $\mathbf{R}$ is the unit vector along the positive normal to the orbital plane, the equations of motion has the form:

$$\dot{\mathbf{h}} = \mathbf{h} \times \frac{\partial V^*}{\partial \mathbf{h}} + \mathbf{e} \times \frac{\partial V^*}{\partial \mathbf{e}}, \qquad \dot{\mathbf{e}} = \mathbf{e} \times \frac{\partial V^*}{\partial \mathbf{h}} + \mathbf{h} \times \frac{\partial V^*}{\partial \mathbf{e}},$$

where V^* is the potential averaged over the mean anomalies.

- Second equation is identically satisfied with $e = 0$. Assuming that the orbit is circular first equation becomes:

$$\dot{\mathbf{R}} = -\omega_0(\mathbf{R}\cdot\mathbf{R}_0)(\mathbf{R}_0 \times \mathbf{R}) - \omega_1(\mathbf{R}\cdot\mathbf{R}_1)(\mathbf{R}_1 \times \mathbf{R})$$

where $\mathbf{R_0}$ is the unit vector along the normal to the equatorial plane of the asteroid, $\mathbf{R_1}$ is the unit vector along the normal to the orbital plane of the Sun, ω_0 is a constant depending on J_2 and ω_1 a constant depending on the mass of the Sun.

• The problem admits 3 equlibrium points in the domain $i \in [0^0, 90^0]$, $\Omega \in [0^0, 180^0]$, two stable and one unstable. The above vectorial equation can be reduced to a set of equations similar to the Euler equations for the motion of a rigid body. The location of equilibria, the amplitude and the period of librations are provided by analytical formulas.

• We performed a study to identify the parameters that lead to largest librations and find out that the obliquity is one of the key parameters. We considered the case of Vesta (obliquity 16 deg according to Bills And Nimmo (2011)) and Eros (obliquity 89 deg according to Souchay et al. (2003)).

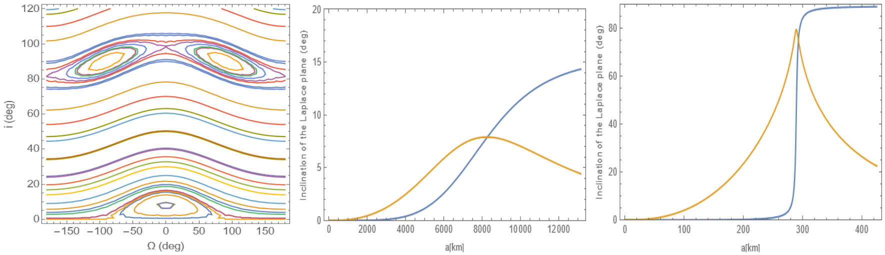
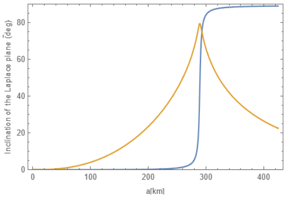
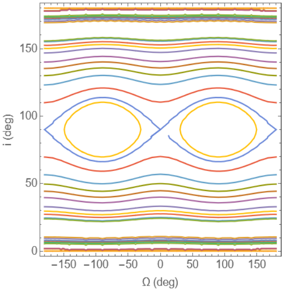


Figure 8:

EROS case a=200 km

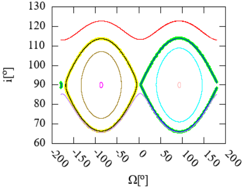


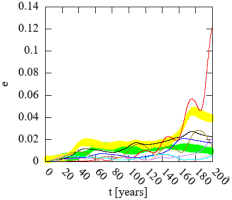


Orbital elements evolution: Omega vs i (left) , e (right)

Initial conditions: a=200 km , e=0.0001

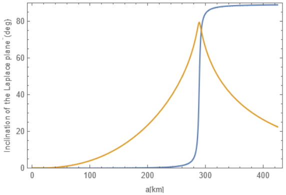
Thick lines: Cartesian approach, Thin lines: Hamiltonian formalism

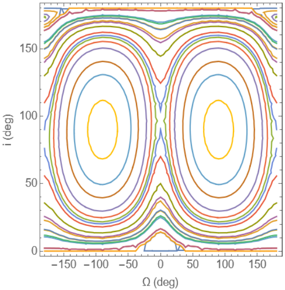




EROS case

a=280 km

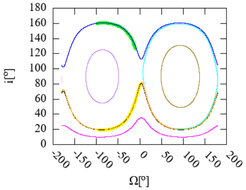


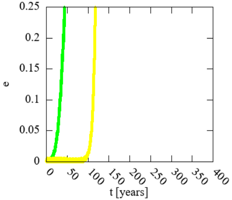


Orbital elements evolution: Omega vs i (left), e (right)

Initial conditions: a=280 km , e=0.000001

Thick lines: Cartesian approach, Thin lines: Hamiltonian formalism





Conclusions

- Development of Cartesian and Hamiltonian models to analyse the dynamics on moderate and long time scales;
- Identify the effects induced by the perturbing terms; Classify resonances and analyse the effects induced by each type of resonance;
- The Hamiltonian models and methods allow to classify and study resonances.
- On long time scales, the coupling between various effects can lead to a complex dynamics.
- From practical perspectives, the study of dynamics around minor bodies might assess the possibility of existence of accompanying swarms of particles or to find solutions (stable trajectories) for spacecraft exploring minor bodies.

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